AESB 2320 Part I Exam 11 March 2015

1. a) This problem follows the falling film derivation in BSL Sect 2.2 up thru Eq. 2.2-9 (BSL 1st ed.) [2.2-11 in BSL 2nd ed.]. It's OK, but unnecessary, to redo that derivation on the exam.

$$\tau_{xz} = (\rho g \cos\beta)x + C_1$$

We have no B.C. on τ, so plug in Newton's law:

$$-\mu \frac{dv_z}{dx} = \rho g \cos\beta\, x + C_1$$

$$\frac{dv_z}{dx} = -\frac{\rho g \cos\beta\, x}{\mu} - \frac{C_1}{\mu}$$

$$v_z = -\frac{\rho g \cos\beta\, x^2}{2\mu} - \frac{C_1}{\mu} x + C_2$$

BC 1: $v_z = 0$ at $x = 0$; $\rightarrow C_2 = 0$

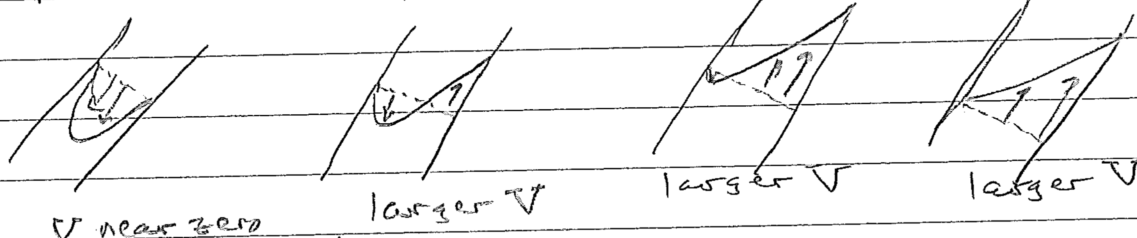
BC 2: $v_z = V$ at $x = \delta$; $V = -\frac{\rho g \cos\beta}{2\mu}\delta^2 - \frac{C_1}{\mu}\delta$

$$C_1 = -\left[V + \frac{\rho g \cos\beta}{2\mu}\delta^2\right]\frac{\mu}{\delta}$$

$$\rightarrow v_z = -\frac{\rho g \cos\beta\, x^2}{2\mu} - \frac{x}{\mu}\left[-\frac{\mu}{\delta}\left(V + \frac{\rho g \cos\beta}{2\mu}\delta^2\right)\right]$$

$$v_z = \frac{x}{\delta}V + \frac{\rho g \cos\beta}{2\mu}\delta^2\left[\frac{x}{\delta} - \left(\frac{x}{\delta}\right)^2\right] \quad \boxed{I}$$

b) If gravity pulls some fluid down, it does so near the top plate, (But at top plate, $v_z = 0$).



So if $\frac{dv_z}{dx} > 0$* at $x = 0$, there is some flow downward.

from Eq. I, $\frac{dv_z}{dx} = \frac{V}{\delta} + \frac{\rho g \cos\beta}{2\mu}\delta^2\left[\frac{1}{\delta} - \frac{2x}{\delta^2}\right]$

at $x = 0$, $\left.\frac{dv_z}{dx}\right|_{x=0} = \frac{V}{\delta} + \frac{\rho g \cos\beta}{2\mu}\delta$

all flow is up if $\left.\frac{dv_z}{dx}\right|_{x=0} = \frac{V}{\delta} + \frac{\rho g \cos\beta}{2\mu}\delta < 0$

or $V < -\frac{\rho g \cos\beta}{2\mu}\delta^2$

* For a Newtonian fluid, this derivation is equivalent to solving for $\tau = 0$ at $x = 0$

2. a) at top of bed: $p_1 = 1\,atm + (0.5)(1000)(9.8)$

" bottom " : $p_2 = 1\,atm + (0.4)(1000)(9.8) - (0.6)(1000)(9.8)$ *

$= 1\,atm - (0.2)(1000)(9.8)$

* p at bottom of tube = $1\,atm + (0.4)(1000)(9.8)$. Since there is no dissipation in tube, p at top of tube is as given above

$$\Delta \mathcal{P} = \frac{\{1\,atm + 0.5(1000)(9.8) - (1\,atm - (0.2)(1000)(9.8))\} + 0.3(1000)9.8}{0.3}$$

$$= \frac{(1)(1000)9.8}{0.3} = 32\,667 \text{ Pa/m}$$

b) We don't know Re, so Eq. 6.4-12 (BSL 2nd ed.) applies

$$\frac{\Delta \mathcal{P}}{L} = 150\left(\frac{\mu v_0}{D_p^2}\right)\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{7}{4}\left(\frac{\rho v_0^2}{D_p}\right)\frac{1-\varepsilon}{\varepsilon^3}$$

$$32\,667 = 150 \frac{0.001}{(0.005)^2}\frac{(0.6)^2}{(0.4)^3} v_0 + \frac{7}{4}\frac{(1000)}{0.005}\frac{0.6}{(0.4)^3} v_0^2$$

$$32667 = 33750\, v_0 + 3281250\, v_0^2$$

$3281250 v_0^2 + 33750 v_0 - 32667 = 0$. Quadratic eq.

$$v_0 = \frac{-33750 \pm [(33750)^2 - 4(3281250)(-32667)]^{1/2}}{2(3281250)} = \frac{-33750 \pm 655600}{13124000}$$

positive root applies: $v_0 = 0.0947 = \frac{Q}{A}$

$Q = (0.0947)\,\pi\,\frac{(0.1)^2}{4} = 0.000744 \text{ m}^3/\text{s}$ (0.74 liter/s)

Note $Re = \frac{D_p \rho v_0}{\mu}\frac{1}{1-\varepsilon} = \frac{(0.005)(1000)(0.0947)}{0.001}\frac{1}{0.6} = 789$

Darcy's law does not apply.

~~(level 1)~~

3. One can do this problem two ways: a) take the inlet at the bottom of the tank, just above the abrupt constriction. The p_1 must account for the weight of 2 m of water above. b) Take level 1 at top of tank.* That is what I do here.

Eq. 7.5-10 applies. Let's go term by term:

$v_1 \cong 0 \quad V_2 = V^2$ (V = velocity in pipe)	$\frac{1}{2}V^2$
$z_2 - z_1 = -(23+2) = -25$ m.	$-25(9.8)$
$\frac{p_2 - p_1}{\rho} = \frac{1\,atm - 1\,atm}{\rho} = 0$	0
$\hat{W}_m = 0$ No work in or out	0
pipe: $L = 10 + 15 + 20 = 45$; $R_h = D/4 = 0.05$	$-\frac{1}{2}V^2 \frac{45}{0.05} f$
fittings (see table 7.5-1)	
sudden contraction @ bottom of tank	$-\frac{1}{2}V^2(0.45)$
2 sharp elbows 90°; take middle value	$-\frac{1}{2}V^2(1.6)\times 2$
(any value between 1.3 and 1.9 is OK.)	

$$\frac{1}{2}V^2 - 25(9.8) + 0 = 0 - \frac{1}{2}V^2\frac{55}{0.05}f - \frac{1}{2}V^2(3.65) \quad \text{rearranging}$$

$$245 = V^2\left[\frac{1}{2} + \frac{1}{2}\frac{45}{0.05}f + \frac{3.65}{2}\right] = \left[2.325 + 450f\right]V^2 \quad \boxed{\text{Eq. I}} \text{ (see over)}$$

b) Since we don't know V, we don't know Re or f. But as suggested we start by assuming Re very large. $\frac{k}{D} = \frac{0.0008}{0.2} = 0.004$. As long as Re $\gtrsim 3\cdot 10^5$, $f \approx 0.007$. (Fig 6.2.2, 2nd ed. BSL.

$$245 = \left[2.325 + 450(0.007)\right]V^2 = \left[2.325 + 3.15\right]V^2 = 5.525\,V^2$$

$$6.7 = V \qquad [6.7\text{ m/s}]$$

Need to check Re + f, + maybe repeat.

$$Re = \frac{(0.2)6.7(1000)}{0.001} = 1.3\cdot 10^6.$$ Assumption was good.

We're done.

[The pipe could carry $\pi(0.1)^2\, 6.7 = 0.21\ m^3/s$ water.]

SEE MORE ON NEXT PAGE

*If one takes level 1 as bottom of tank, $z_2 - z_1 = -23$ m, and $\frac{p_2 - p_1}{\rho} = 2(9.8)$. Same final result.

4. This is a falling film, again. This time the derivation in Sect 2.2 applies up thru Eq. (BSL 1st ed.) [Eq. 2.2-13 in 2nd ed.].

$$\tau_{xz} = \rho g \cos\beta\, x = 1000(9.8)(0.00349)\,x$$
$$= 34.2\,x$$

Flow occurs if $\tau_{xz} > \tau_0$ anywhere. τ_{xz} has its largest value at $x = 1$ m (the bottom). Rocky needs

$$\tau_0 > 34.2 \text{ Pa}$$

More on problem 3:

More generally, because we don't know V or Re, we would solve for V by iteration. Eq. I on previous page could be rearranged:

$$V = \left(\frac{245}{2.325 + 450 f}\right)^{1/2}$$

This eq. plays the role of Eq. 6.1-4 in the trial-and-error process.* Eq. 6.1-4 is based on a momentum balance with no kinetic energy effects, fittings, or work done on or by the system. It therefore does not apply to this problem.

* that is: guess V, calc Re, find f on chart, calculate V from Eq. 1, repeat.